$\theta = \pi/3$ and our analysis gives the allowed region described by: $3ab - a - b - 1 \geq 0$. To ascertain which θ is the physical one, we insist that the time duration between collisions are all positive. The formulas for the time ratios here are:

$$t_{2j}/t_{2j-1} \;=\; -\cos((j-1/2)\theta)/\cos((j+1/2)\theta) \text{ and} \tag{100}$$

$$t_{2j+1}/t_{2j} \;=\; -\sin(j\theta)/\sin((j+1)\theta). \tag{101}$$

These must be nonnegative for $j \leq n-1$, which requirement holds only for $(n-1)\pi/n \leq \theta \leq \pi$. The only suitable θ from Eq. (98) is $(2n-1)\pi/(2n+1)$ and so the allowed region for $(31)^n$ is given by $ab\cot^2((2n-1)\pi/(2(2n+1))) - a - b - 1 \geq 0$. That is, it lies to the right of the right branch of the hyperbola given by:

$$ab\cot^2\left(\frac{(2n-1)\pi}{2(2n+1)}\right) - a - b - 1 = 0. \tag{102}$$